Substituting Eqs. (14) and (5) into Eq. (12), we obtain the following formula for x_c:

$$x_c = \frac{3}{4}\left(\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i x_i^c S_i\right) \Big/ \left(\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i S_i\right) \tag{15}$$

The centroid position vector can be written in the following vector form:

$$\boldsymbol{r}_c = \frac{3}{4}\left[\sum_{i=1}^{N} \left(\boldsymbol{r}_i^c \cdot \boldsymbol{n}_i\right)\boldsymbol{r}_i^c S_i\right] \Big/ \left[\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i S_i\right] \tag{16}$$

Conclusions

A new formulation for the centroid of an arbitrary polyhedron bounded by arbitrary planar polygonal faces is derived. Compared with an earlier formulation, the new formula does not need numerical integrations using Gaussian quadratures over the faces. Only face properties such as areas, unit normals, and face centers are employed to compute the cell centroid. The implementation of the new formula is, therefore, straightforward. The extension of the formula to polyhedra bounded by nonplanar polygons can be accomplished by decomposing the nonplanar polygons into planar ones. Therefore, the formulation can be used to compute geometric properties for arbitrary polyhedra.

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Euler/Navier–Stokes Optimization of Supersonic Wing Design Based on Evolutionary Algorithm

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Introduction

APPLICATION of numerical optimization to aerodynamic design is a difficult task. In Ref. 1, it was reported that the distribution of the objective function could be extremely multimodal even in a simplified problem. In addition, function evaluations using a computational fluid dynamics (CFD) code, especially an Euler or Navier–Stokes code, are very expensive. Therefore, both an optimization algorithm with high parallel efficiency and a powerful parallel computer are required to accomplish aerodynamic optimization.

Among optimization algorithms, evolutionary algorithms[2] (EAs) appeal to many designers and researchers because of their robustness. High parallel efficiency is also achieved by using a simple master–slave concept for function evaluations if such evaluations consume most of the CPU time. Aerodynamic optimization using CFD will be used as a typical case.

The purpose of this study is to examine the feasibility of supersonic wing design optimization using EAs coupled with Euler/Navier–Stokes computation. To overcome the expected difficulty in CPU time, computation is performed on a parallel vector machine called the numerical wind tunnel[3] (NWT). Grid generation and flow calculation of each design candidate are distributed to 64 processing elements (PEs), whereas EA operators are assigned to the master computer because their CPU time is negligible.

Formulation of Optimization Problem

In this study, an aerodynamic shape of a supersonic wing is optimized at the supersonic cruise design point. The cruising Mach number is set to 2.3. The purpose of the present study is to maximize the lift-to-drag ratio L/D, maintaining substantial lift coefficient C_L and wing thickness. The optimization problem is defined as follows: The objective function to be maximized is L/D with the constraints $C_L = 0.1$ and thickness to chord $t/c \geqq 0.35$.

The lift constraint is satisfied by changing the geometric angle of attack at the wing root so that C_L becomes 0.1 based on the lift coefficient varying linearly. This approach requires two extra flow evaluations.

The aerodynamic performance is evaluated by using an Euler/Navier–Stokes code. This code employs total variation diminishing-type upwind differencing,[4] the lower–upper symmetric Gauss–Seidel scheme, and the multigrid method.

Airfoil sections of design candidates are generated by the extended Joukowski transformation.[5] It transforms a circle to various kinds of airfoils by two consecutive conformal mappings using five parameters: x_c, y_c, x_t, y_t, and Δ. Airfoil sections defined by these extended Joukowski parameters and the twist angle will be given at eight span sections; spanwise locations are also treated as design variables except for the wing root and tip locations. Wing geometry is then interpolated in the spanwise direction by using the second-order spline interpolation. The planform is assumed to be a double-delta wing similar to the National Aerospace Laboratory scaled supersonic experimental airplane.

Optimization Using EA

In the present EA, design variables are coded in finite length strings of real numbers corresponding to the five Joukowski transformation parameters, the twist angle, and their spanwise locations. Fitness of an individual is determined by its rank among the population based on its L/D. Selection is performed by the stochastic universal sampling[6] coupled with the elite strategy. Ranking selection is adopted because it maintains sufficient selection pressure throughout the optimization. Then the offspring (the new design candidates) are produced, applying a one-point crossover and an evolutionary direction operator[7] half-and-half to the mating pool (selected design candidates). During the reproduction process, mutation takes place at a probability of 20% and then adds a random disturbance to the corresponding gene. The population size is kept at 64.

To reduce the wall clock time necessary for this optimization, evaluations using the Euler/Navier–Stokes code are distributed to 64 PEs of the NWT. Because the CPU time used for EA operators is negligible, turnaround time becomes almost $\frac{1}{64}$. Actually, whereas each CFD evaluation took about 1 h of CPU time (for three Euler evaluations) on the slave PE, the EA operators took less than 1 s on the master PE.

Results

Because the wing planform is fixed and the viscous drag primary depends on the planform area, inviscid calculations are used for the present evaluations. The total drag evaluated here consists of the

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volume wave drag, the lift-dependent wave drag, and the induced drag. Among the three drag components, the lift-dependent wave drag primary depends on the planform. Therefore, a design that achieves the minimum volume wave drag and the minimum induced drag will ensure the feasibility of the present approach.

The optimization history of the present EA is shown in Fig. 1 in terms of C_D. The design has drag coefficient of 77.7 counts and L/D of 12.83. Because the evaluation takes about 1 h per generation, the optimum is obtained in 50 h.

Figure 2 shows a comparison of the spanwise loading distribution of the designed wing with a parabola that is known to give the minimum induced drag when the structural constraint is considered.[5] The parabolic load distribution indicates that the design achieves the minimum induced drag.

The optimized airfoil sections and the corresponding pressure distributions at the 0, 33, and 66% spanwise locations are shown in Fig. 3. The designed wing increases the camber toward the wing tip to increase C_L. This helps to yield the parabolic load distribution to achieve the minimum induced drag. On the other hand, the airfoil thickness becomes as thin as possible in the given design space to minimize the volume wave drag, as expected. The plot is not shown here because the thickness is simply 3.5% of the chord.

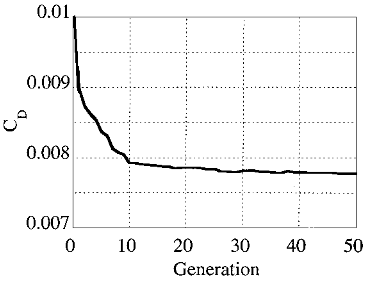

Fig. 1 Optimization history.

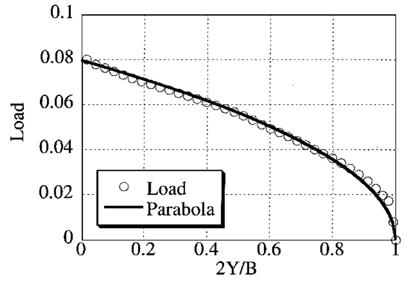

Fig. 2 Spanwise load distribution of the designed wing.

Conclusions

An EA coupled with an Euler/Navier–Stokes code has been applied to supersonic wing shape design. To overcome the enormous computational time necessary for the optimization, aerodynamic evaluations are distributed to the PEs of NWT. Parallelization of EA on NWT is straightforward, and its performance is extremely good in reducing the turnaround time.

The optimum design obtained from the present approach yields both the minimum induced drag and the minimum volume wave drag in the given design space. This indicates the feasibility of the present approach for aerodynamic design of supersonic transport.

In addition, the present study indicates the most important features of supersonic wing design as compared with conventional transonic wing design as follows:

1) Warp geometry based on camber line and twist angle distributions plays a more important role than thickness distribution.

2) The structural constraint is found to be important to determine wing thickness, and thus, a more practical structural constraint will be required.

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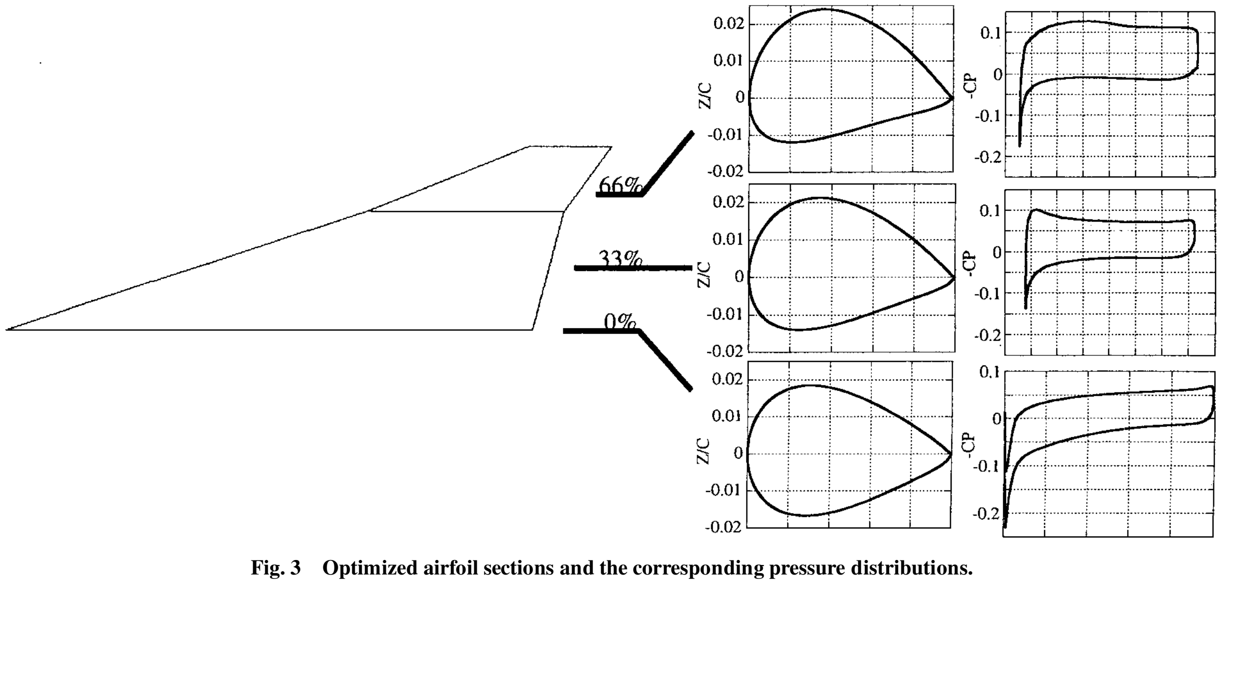

Fig. 3 Optimized airfoil sections and the corresponding pressure distributions.